

Mixed Integer Multi-Objective Goal Programming Model For Green Capacitated Vehicle Routing Problem

Adibah Shuib^{1*}, Nurul Asma Muhamad²

¹Centre for Mathematics Studies, Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA (UiTM), 40450 Shah Alam, Selangor, Malaysia. ²Malaysia Institute of Transport (MITRANS), Universiti Teknologi MARA (UiTM), 40450 Shah Alam, Selangor, Malaysia.

*Corresponding Author: adibah@tmsk.uitm.edu.my

Abstract

Nowadays, environmental effects of logistics and extensive consumption of natural resources have been given more attention by governments and industries. Conventionally, these issues and their impact on distribution logistics were not emphasized specifically or addressed directly when solving the Vehicle Routing Problem (VRP). Hence, Green VRP (GVRP) has been introduced to take into consideration both the economic and environmental costs when determining effective routes for distribution services. GVRP is a branch of green logistics in which the externalities of using vehicles, enhancement of transportation effectiveness at operational level, ensuring optimal energy consumption of vehicles, and minimizing fuel consumption are taken into account in the routing and scheduling. This paper presents our study which concerns with formulating a mathematical programming model of Green Capacitated VRP (GCVRP), which focuses are on minimization of the greenhouse gas emissions and fuel consumption, to assuage the resulting effects of transportation on the environment. The formulated Mixed Integer Goal Programming (MIGP) model has multiple objective functions as its goal, which are minimizing the total distance travelled, minimizing the total fuel consumption and minimizing the total Carbon Dioxide emissions. Two set of benchmark instances have been used to test the proposed model. The MIGP model is solved by the preemptive GP approach and using the MATLAB intlinprog solver. Based on the computational results, the model has been proven to be able to produce optimal solutions, thus indicating that it has the potential to be applied to real-world VRPs.

Keywords: green logistics, green capacitated vehicle routing problem, mixed integer goal programming model, preemptive method, fuel consumption, carbon dioxide emission

Introduction

Today, governments and businesses are more concerned about green logistics. The term green logistics refers to more sustainable ways of managing

logistics such that its environmental, economics and social effects can be seriously considered. Greater attentions are given to the environmental effects and the indiscriminate use of natural resources in logistics, in which matters such as the fuel consumption, air pollution, and Greenhouse Gas (GHG) emission, are taking center stage. Other concerns include sustainability, awareness towards clean environment, and on protection and conservation of environment (Thiery, Salomon, Van Nunen, & Van Wassenhove, 1995) aside from logistics methods which result in cost savings and higher benefits environmentally and economicwise to organizations (Sbihi & Eglese, 2010).

Vehicle Routing Problem (VRP) plays vital role in distribution management. It has been at the core of many operations research problems and applied in variety of industrial applications involving delivery and collection services of goods or people. VRP is concerned with determining a set of optimal routes at a minimal cost for a fleet of m vehicles, which begin and end the routes at the depot, when serving a group of n customers or users who are geographically dispersed such that the known demand of all customers can be satisfied. In addition, each customer can only be visited by exactly one vehicle. When certain vehicle capacity constraint is imposed, the variant of VRP is known as the Capacitated Vehicle Routing Problem or CVRP (Toth & Vigo, 2002).

Meanwhile, the Green VRP (GVRP) refers to a sub-problem of green logistics whereby related factors such as the GHGs emissions and fuel consumptions are being considered. According to Lin, Choy, Ho, Chung, and Lam (2014), GVRP involves reduction in fuel amount used per trip, optimal utilization of energy for the vehicles and enhanced transportation effectiveness at operational level. Increased competition in logistics industries is making GVRP more attractive and shifting towards a more sustainable transportation that has less adverse impacts toward the environment and ecology becoming vital (Yasin & Yu, 2013). Thus, GVRP focuses on minimizing the environmental effects as opposed to merely just minimising costs and total distance traveled. Optimal solution of GVRP is composed of routes with less total traveling time and total distance to

achieve green routes, that enable reduced fuel consumptions and GHG emission. Among the GHGs, CO₂ has the major share in a global basis due to its emission, primarily through burning of carbon fuels such as oil, gas, and coal, used in transportation and power (energy) generation.

Based on past studies on Green CVRP (GCVRP), the mathematical programming models used include Integer Programming or IP (El Bouzekri El Idrissi et al., 2013, 2016; Xiao et al., 2012), Binary Integer Programming (BIP) model (Ubeda et al., 2014), and Mixed Integer Programming (MIP) model (Küçükoğlu et al., 2013). In studies by Ubeda et al. (2014) and El Bouzekri El Idrissi et al. (2013), minimizing the total CO₂ emission of vehicles has been the goal of the model, while in Xiao et al. (2012) and (Küçükoğlu et al., 2013), the objective function concerns with minimizing the amount of fuel consumed subjected to the distance travelled and vehicle load. These past studies formulated mathematical programming models with single objective function. On the other hand, El Bouzekri El Idrissi et al. (2016) formulated the model as a multi-objective GVRP in which the objective functions are to minimize the total costs and the overall CO₂ emissions.

Despite many researches have been carried out on GCVRP, there are still many opportunities for new studies and methods since the green logistics itself is a vast and relatively new topic. Meanwhile, studies in GCVRP that aim at minimizing the fuel consumption and CO₂ emission simultaneously are still few. This paper intends to present the mathematical programming model formulated in our study. The proposed Mixed Integer Multi-Objective Goal Programming (MIMOGP) model has been used to determine the optimal solution of GCVRP based on selected test instances. The goals of the model include minimizing the total distance travelled, minimizing the total fuel consumed, and minimizing the total CO₂ emitted. This paper is organized in four main sections: Introduction, Method, Results and Discussion, and the Conclusion.

Method

There are four phases in our study. Phase 1 deals with preliminary works, which include analyzing benchmark instances and factors affecting fuel consumption and CO₂ emissions. Phase 2 involves formulating the objective functions and constraints of the model. Phase 3 concerns with solution method, which comprises of setting up the matrices required as input for the MATLAB *Intlinprog* solver. The final phase, Phase 4, is designated to solving the model based on the preemptive GP approach and analyzing the results.

As described in Taha (2007), preemptive approach and weights method can be used to solve GP models. Weights method refers to an approach where a single objective function is formulated as weighted sum that relates the goals of the model whereas the preemptive approach involves deciding the order of priority of the goals. In preemptive approach, the model's optimal solution is found by repetitively solving the model according to a goal each time, starting with the highest priority goal. In each time, the optimal solution obtained is inserted as additional constraint prior to executing the next goal. In this study, the MIMOGP model for GCVRP is solved using the preemptive GP approach.

a. Data

Due to the limitation in getting the real data due to companies policies, CVRP benchmark instances involving the Set A and Set B from Augerat et al. (1995) have been used for computational experiments using the proposed model. Set A comprises of 27 instances in which the number of customers (n) of a test instance ranges from 32 to 80 customers. In addition, demand of customer i (d_i) varies, ranging from one (1) unit to 30 units, whilst the capacity of vehicles (Q) is fixed with $Q = 100$ units. Customers in Set A are positioned randomly in the square of $[1,100] \times [1,100]$ while the depot of each instance is also random. On the other hand, 23 instances in Set B have n ranges from 31 to 78, $Q = 100$ units, and d_i takes a value of one unit up to 30 units. The depot in Set B instances is set

randomly while the customers are positioned in clusters (clustered customers). In our study, instances from Set A and Set B have been used in solving the proposed model. These instances are A-n32-k5 and B-n31-k5. For each data, only 10 customers have been selected, aside from the depot, as shown in Figure 1 and Figure 2.

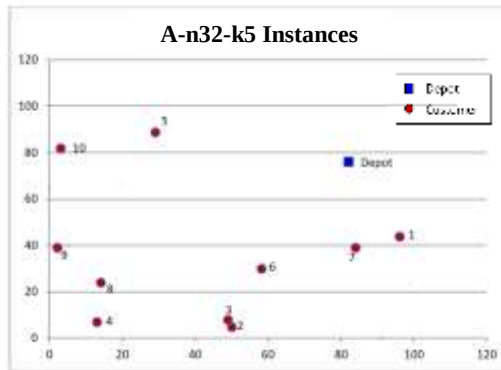


Figure 1: Location of Depot and 10 Customers of A-n32-k5 Instances

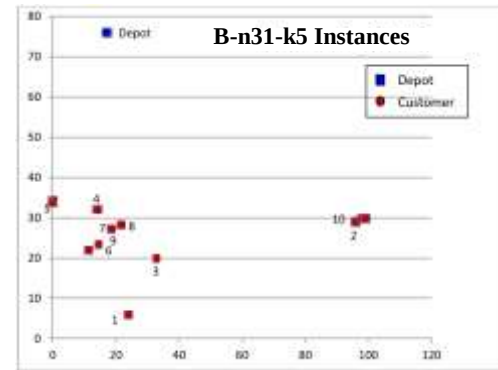


Figure 2: Location of Depot and 10 Customers of B-n31-k5 Instances

b. Model Formulation

Prior to model formulation, model's assumptions were identified first. These assumptions are as follows: i) vehicles are homogeneous; ii) the fleet size is pre-determined; iii) vehicle route starts and ends at a depot; iv) each customer is served only once by one vehicle; v) demand at each node is known and deterministic; v) total customers' demands per vehicle must not exceed the vehicle's capacity; and vi) the road network is an oriented graph.

Objective Function 1 (Goal 1): to minimize total travel distance (TD)

Consider a road network represented as a graph $G(V, A)$ in which V refers to the set of nodes in the network, where $V = \{0, 1, 2, \dots, N\}$ while A denotes the set of edges from i to j where $i, j = 1, 2, \dots, N, i \neq j$. The node 0 is the depot while the total number of customers is N and the total number of nodes is $N + 1$. Each customer is associated to a positive demand. The first objective function of the MIMOGP model for CVRP in this study aims at finding routes with minimum

distance to serve customers using a fleet of K vehicles available in a single depot. The objective function is as follows:

$$\text{Minimize } TD = \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^K c_{ij} x_{ijk} \text{ where } i, j = 1, 2, \dots, N, i \neq j \quad (1)$$

where c_{ij} be the Euclidean distance between customer i to customer j and x_{ijk} denotes the binary decision variables such that $x_{ijk} = 1$ if a vehicle k travels from from node i to node j , and 0, otherwise.

Objective Function 2 (Goal 2): to minimize the total fuel consumption (TFC) related to distance and vehicle load

The second objective function for the MIMOGP model concerns with total fuel consumption. The coefficients of this objective function are determined using the regression equation of Huang et al (2012) where the fuel consumption is proportional to distance and vehicle load, as follows:

$$\text{Fuel consumption} = a_1 (\text{distance}) + a_2 (\text{vehicle load}) + b \quad (2)$$

The values of a_1 , a_2 and b (the regression coefficients) of the regression equation are found using a set of fuel consumption data with various vehicle loads and different distances of Li, Chen and Yao (2010). Based on the regression analysis conducted, the values of a_1 , a_2 and b , are 0.3403, 0.3043 and -8.8515, respectively. The objective function that minimizes the total fuel consumption (TFC) is given in Equation (3), which is adapted from Küçükoğlu et al. (2014). This equation considers three factors, the vehicle vehicle load, the total distance of route, and the vehicle's technical specifications, while values of a_1 , a_2 and b are as determined earlier.

$$\begin{aligned} \text{Minimize } TFC = & \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^K a_1 c_{ij} x_{ijk} + K a_2 q_0 + \sum_{i=1}^N a_2 q_i \\ & + \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^K b x_{ijk} \end{aligned} \quad (3)$$

in which q_i represents the current vehicle load at customer i whereas q_0 is the initial load at the depot.

Objective Function 3 (Goal 3): to minimize Carbon Dioxide emissions

Minimizing the total Carbon Dioxide (CO₂) emitted is the third objective function considered for the GCVRP MIMOGP model of this study. The formulation of this function, which is based on El Bouzekri et al. (2013), is as the following:

$$\text{Minimize } TCE = \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^K e_{ij} x_{ijk} \quad , i, j = 1, 2, \dots, N, i \neq j \quad (4)$$

where e_{ij} denotes the linearized flow of CO₂ emission from node i to node j , which can be represented as an emission matrix for arc $i \rightarrow j$ based on q , the load of vehicle (in ton), and c , the distance from i to j . Equation (5) describes the equation for e_{ij} .

$$e_{ij}(q, c) = c_{ij} \left[\left(\frac{e_{fl} - e_{el}}{Q} \right) q_{ij} + e_{el} \right] \quad (5)$$

In Equation (5), e_{el} denotes the CO₂ emissions of an empty vehicle whereas e_{fl} is the CO₂ emissions of a fully loaded vehicle (by weight). Volume capacity of a vehicle is denoted by the variable Q while c_{ij} represents the symmetrical distance from node i to node j and q_{ij} is the load (in ton) carried from

the node i to node j . In our study, the CO₂ emission of an empty vehicle, e_{el} , and a fully loaded (by weight) vehicle, e_{fl} , for Heavy Duty Vehicle (HDV) are 1.096 kg km⁻¹ and 0.772 kg km⁻¹, respectively, based on El Bouzekri (2013a).

c. Multi-Objective MIGP Model

The following is formulation of the MIGP model for the GCVRP.

Model Notation

$G = (V, A)$	: a complete and strongly connected graph
V	: set of n nodes where $V = \{0, 1, 2, \dots, N\}$
A	: set of arcs from i to j where $i, j = 1, 2, \dots, N, i \neq j$
S	: set of customers, $S = \{1, 2, \dots, N\}$,
H	: set of vehicles, $H = \{1, 2, \dots, K\}$
N	: number of nodes in V
K	: number of available vehicles
i, j	: node index, $i, j = 1, 2, \dots, N$
k	: vehicle index, $k = 1, 2, \dots, K$
TD	: total distance travelled by vehicles
TFC	: total fuel consumed by vehicles
TCE	: total CO ₂ emitted by vehicles
a_1, a_2 and b	: coefficients of regression equation
d_i	: demand of customer i
Q_k	: capacity of vehicle k
c_{ij}	: distance from node i to node j .
e_{ij}	: CO ₂ emission between node i to node j .
Min_load	: weight of an empty vehicle
Max_load	: maximum load of vehicle which is $Min_load + Q_k$

Decision Variables:

x_{ijk}	= $\begin{cases} 1, & \text{if a vehicle } k \text{ travels from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$
z_{ik}	= $\begin{cases} 1, & \text{if customer } i \text{ is visited by vehicle } k \\ 0, & \text{otherwise} \end{cases}$
q_i	: vehicle's load at customer i

Objective functions:

$$\text{Minimize } TD = \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^K c_{ij} x_{ijk} \quad , \quad i \neq j \quad (6.1)$$

$$\begin{aligned} \text{Minimize } TFC = & \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^K a_1 c_{ij} x_{ijk} + K a_2 q_0 + \sum_{i=1}^N a_2 q_i \\ & + \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^K b x_{ijk} \quad , \quad i \neq j \end{aligned} \quad (6.2)$$

$$\text{Minimize } TCE = \sum_{i=0}^N \sum_{j=0}^N \sum_{k=1}^K e_{ij} x_{ijk} \quad , \quad i \neq j \quad (6.3)$$

subject to:

$$\sum_{i=1}^N x_{i0k} = 1 \quad k = 1, 2, \dots, K \quad (6.4)$$

$$\sum_{i=1}^N x_{0ik} = 1 \quad k = 1, 2, \dots, K \quad (6.5)$$

$$\sum_{k=1}^K z_{ik} = 1 \quad i = 1, 2, \dots, N \quad (6.6)$$

$$\sum_{i=1}^N d_i z_{ik} \leq Q_k \quad k = 1, 2, \dots, K \quad (6.7)$$

$$\sum_{i=0}^N x_{ijk} - \sum_{i=0}^N x_{jik} = 0 \quad j = 1, 2, \dots, N; \quad i \neq j; \quad k = 1, 2, \dots, K \quad (6.8)$$

$$\sum_{i=0}^N \sum_{k=1}^K x_{ijk} = 1 \quad j = 1, 2, \dots, N; \quad i \neq j \quad (6.9)$$

$$\sum_{j=0}^N \sum_{k=1}^K x_{ijk} = 1 \quad i = 1, 2, \dots, N; \quad i \neq j \quad (6.10)$$

$$q_0 = \text{Min_load} \quad (6.11)$$

$$q_i - q_j \geq d_i - M \left(1 - \sum_{k=1}^K x_{ijk} \right) = 0 \quad i, j = 1, 2, \dots, N; \quad i \neq j \quad (6.12)$$

$$\text{Min_load} \leq q_i \quad i = 1, 2, \dots, N \quad (6.13)$$

$$q_i \leq \text{Max_load} \quad i = 1, 2, \dots, N \quad (6.14)$$

$$x_{ijk} \in \{0, 1\}, \quad z_{ik} \in \{0, 1\} \quad i, j = 1, 2, \dots, N; \quad k = 1, 2, \dots, K \quad (6.15)$$

$$q_i \geq 0 \quad i = 0, 1, 2, \dots, N \quad (6.16)$$

The model's objective functions are described in Equations (6.1), (6.2) and (6.3), which are to minimize the total travel distance, total fuel consumption, and the total CO₂ emissions, respectively. Constraints (6.4), (6.5) dictate that all vehicles begin and complete their routes at the depot. Constraint (6.6) is imposed to ensure that each node, other than depot, is served by exactly one vehicle. Meanwhile, Constraint (6.7) guarantees that capacity of vehicle is not exceeded while Constraint (6.8) indicates that a vehicle that arrives at a customer must also depart from that customer. Constraint (6.9) and (6.10) guarantee all customers are served. Constraint (6.11) imposes that vehicles are empty at the depot. Constraint (6.12) is included to avoid the sub tours. Constraints (6.13) and (6.14) restrict the load of the vehicle at a visited customer. Constraint (6.15) allows only binary integer values for decision variable x_{ijk} and z_{ik} . Constraint (6.15) represents the non-negativity restriction on q_i , the vehicle load at customer i .

Results and Discussion

The multi-objective MIGP model for GCVRP was solved using the MATLAB *intlinprog* solver and preemptive GP approach. Computational results for A-n32-k5 instances are as shown in Table 2. Note that the MIGP model is first solved based on Goal 1 where the minimum TD obtained was 361.60 units. This result is added as additional constraint to the model that restricts the total distance traveled by the vehicles not to exceed 361.60 units. Then the MIGP model is solved based on Goal 2. The minimum TFC is 165.6373 liters with total load of the vehicle and minimum TD achieved remain the same. Next, additional constraint that restricts the TFC to be less than or equal to 165.6373 liters is added to the model. The MIGP model is then solved using Goal 3. Based on Table 2, the minimum TEC obtained is 373.6468 kg CO₂. Similar steps have been taken when solving the MIGP model using the B-n31-k5 instance and results are summarized

in Table 3. Figure 3 and Figure 4 show the optimal routes for A-n32-k5 and B-n31-k5 instances, respectively.

Table 2: Summary of Results for A-n32-k5 Instances

Goals	Veh.	Route	Total Distance, c_{ij} (units)	Total Vehicle load (units)	Total Fuel Consumption (Liter)	CO ₂ Emissions (kg CO ₂)
Min	1	[0 7 1 0]				
TD	2	[0 5 10 9 8 4 2 3 6 0]	361.60	130	-	-
Min	1	[0 5 10 9 8 4 2 3 6 0]				
TFC	2	[0 7 1 0]	361.60	130	165.64	-
Min	1	[0 5 10 9 8 4 2 3 6 0]				
CO ₂	2	[0 7 1 0]	361.60	130	165.64	373.65

Note: Veh. - Vehicle. e.g. [0 7 1 0] means route is 0 - 7 - 10 - 0 and '0' is the depot.

Based on Table 2 and Table 3, TD for A-n32-k5 instance is higher than that of B-n31-k5 instance since more vehicles are used and extra distance encountered when going back and forth to depot. However, the TFC and TEC obtained when the model is tested using B-n31-k5 instance are less as compared to those of A-n32-k5. This is because the demand or load per route (i.e., per vehicle) is lower for each route in the case of B-n31-k5 instance, thus, less fuel is consumed and less CO₂ is emitted

Table 3: Summary of Results for B-n31-k5 Instances

Goals	Vehicle	Route	Total Distance, c_{ij} (units)	Total Vehicle load (units)	Total Fuel Consumption (Liter)	CO ₂ Emissions (kg CO ₂)
Min	1	[0 5 1 3 0]				
TD	2	[0 10 2 0]	533.52	139	-	-
	3	[0 8 7 9 6 4 0]				
Min	1	[0 8 7 9 6 4 0]				
TFC	2	[0 5 1 3 0]	533.52	139	116.85	-
	3	[0 10 2 0]				
Min	1	[0 8 7 9 6 4 0]				
CO ₂	2	[0 10 2 0]	533.52	139	116.85	442.1690
	3	[0 5 1 3 0]				

Note: e.g. [0 8 7 9 6 4 0] means route is 0- 8 - 7 - 9 - 6 - 4 - 0 and '0' is depot.

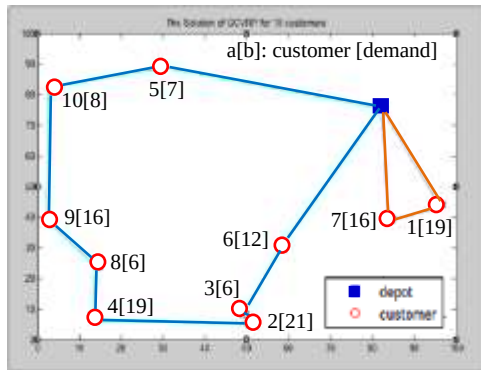


Figure 3: GCVRP Optimal Solution for A-n32-k5 Instances

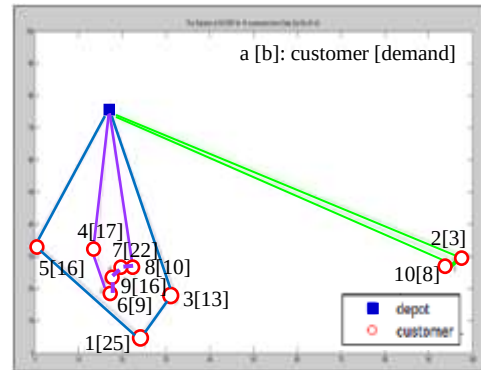


Figure 4: GCVRP Optimal Solution for B-n31-k5 Instances

Conclusion

The results obtained have proven that the multi-objective MIGO model for the GCVRP formulated is capable of producing optimal solutions for the small size problem of 10 customers such that the total distance traveled, the total fuel consumption and the total CO₂ emission are minimized. Thus, the proposed MIMOGP model of this study has the potential to be applied to real-world GCVRP problem and for determining optimal routes that produce significant cost savings and higher economic and environmental benefits.

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